Imaginary Potential Induced Quantum Coherence for Bose-Einstein Condensates

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The role of complex potentials in single-body Schrődinger equation has been studied intensively. We study the quantum coherence for degenerate Bose gases in complex potentials, when the exchange symmetry of identical bosons is considered. For initially independent Bose-Einstein condensates, it is shown that even very weak imaginary potential can induce perfect quantum coherence between different condensates. The scheme to observe imaginary potential induced quantum coherence is discussed.

The extension of quantum mechanics to complex potentials has been studied intensively in diverse areas of physics [1, 2]. The Hamiltonian of the system with complex potentials is not Hermitian any more. How to understand and reveal the fundamental properties of non-Hermitian Hamiltonians is very interesting, especially after Bender *et al.* [3] found the counter-intuitive result that non-Hermitian Hamiltonians have entirely real eigenvalue spectra for some physical systems satisfying the parity-time symmetry. Besides an alternative formulation of quantum mechanics [1] with complex potentials, it has potential applications in a lot of different systems, such as molecular collisions [2], matter wave dynamics [4, 5, 6, 7, 8], light propagation [9, 10, 11], optical solitons [12], and quantum transport [13] *etc.*.

Ultracold atomic gases are promising to study quantum behavior with complex potentials, because various complex potentials can be realized and manipulated experimentally [4, 5, 6, 7, 8]. Although all experiments up to date [4, 5, 6, 7, 8] are not about degenerate gases, the remarkable advances of degenerate Bose and Fermi gases [14] make it feasible to study quantum many-body physics with complex potentials. Quite surprising, to our best knowledge, both theoretical and experimental studies on quantum many-body physics in complex potentials are highly scarce, although the single-body problem has been intensively studied for cold atoms [4, 5, 6, 7, 8, 15, 16]. An important reason for this situation lies in the common belief that the external potential itself will not influence quantum many-body behavior, without considering interatomic interaction. Different from this common belief, our work shows that complex potentials may play an important role in quantum many-body behavior. Based on the many-body Schrődinger equation for identical bosons, we focus our studies on the quantum coherence for two initially independent Bose-Einstein condensates (BECs) in a complex periodic potential. It is found that even a very weak imaginary potential can induce perfect quantum coherence between two condensates. Our theoretical work gives an example that complex potentials not only play a role in single-body problem, but also play important role in quantum many-body behavior. This result opens the way to study novel quantum many-body physics for ultracold gases [19] with complex potentials.

Complex potentials can be generated with the interaction between near resonant laser and a two-level atom with an additional decay channel of the excited state to another state [4, 5, 6, 7, 8]. The complex potential takes the following general form [17]

$$V(\mathbf{r}) = \frac{d_e^2 \mathbf{E}^2(\mathbf{r})}{\hbar(\delta + i\Gamma/2)}. \quad (1)$$

Here, d_e is the dipole matrix element of an atom, while δ is the detuning of the light frequency. Γ is the loss rate of the excited level through the additional decay channel. $\mathbf{E}(\mathbf{r})$ is the electric field of the laser. By varying the detuning, one can vary the potential from a perfect real potential to an imaginary potential. The manipulation of the electric field makes various complex potentials possible such as complex periodic potential.

It has been shown that the solution of the single-particle Schrődinger equation with complex potentials given by (1) agrees well with the experimental results about cold atoms [4, 5, 6, 7, 8]. Here we consider the solution of the many-body Schrődinger equation when both complex potentials and exchange symmetry of identical bosons are considered. To show clearly the imaginary potential induced quantum coherence between initially independent condensates, we consider a process shown in Fig. 1. In Fig. 1a, there are two initially independent condensates described by a Fock state $|N_1, N_2\rangle$, with N_1 and N_2 being the initial particle numbers in two condensates. Initially, there is no overlapping between these two condensates. As shown in Fig. 1b, these two condensates are then allowed to expand after switching on a complex periodic potential.

When the exchange symmetry of identical bosons is considered, the many-body wave function for identical

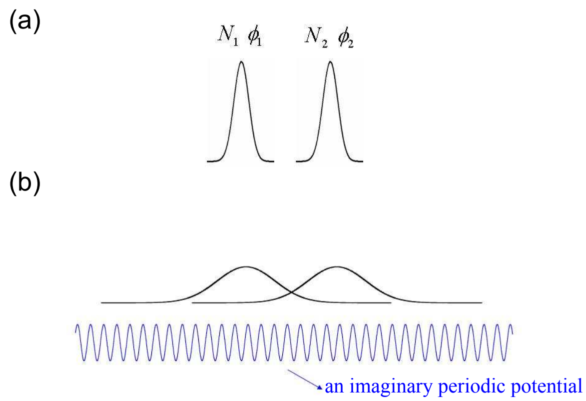

FIG. 1: In Fig. a, there are two initially independent Bose-Einstein condensates. In Fig. b, a complex periodic potential is switched on after these two condensates are allowed to expand.

bosons occupying two modes can be written as

$$\Psi\left(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_N,t\right)=A_n\sqrt{\frac{N_1!N_2!}{(N_1+N_2)!}}$$
$$\sum_P P\left[\phi_1\left(\mathbf{r}_1,t\right)\cdots\phi_1\left(\mathbf{r}_{N_1},t\right)\times\right.$$
$$\left.\phi_2\left(\mathbf{r}_{N_1+1},t\right)\cdots\phi_2\left(\mathbf{r}_{N_1+N_2},t\right)\right], \tag{2}$$

where P denotes $(N_1+N_2)!/N_1!N_2!$ permutations for the bosons in different single-particle wave functions ϕ_1 and ϕ_2. To give a general study, we assume $\zeta(t)=\int\phi_1(\mathbf{r},t)\,\phi_2^*(\mathbf{r},t)\,d\mathbf{r}$ from the beginning to consider the possible nonorthogonality between ϕ_1 and ϕ_2. We assume further $\eta_1(t)=\int|\phi_1(\mathbf{r},t)|^2\,d\mathbf{r}$ and $\eta_2(t)=\int|\phi_2(\mathbf{r},t)|^2\,d\mathbf{r}$ to consider the decay of η_1 and η_2 due to complex potentials. A_n is introduced so that the average overall particle number is $N_1\eta_1+N_2\eta_2$. After lengthy calculations, we have

$$|A_n|^2=\sum_{i=0}^{\min(N_1,N_2)}\frac{N_1!N_2!\eta_1^{N_1-i}\eta_2^{N_2-i}\,|\zeta(t)|^{2i}\,(N_1+N_2)}{i!i!\,(N_1-i)!\,(N_2-i)!\,(N_1\eta_1+N_2\eta_2)}. \tag{3}$$

In this paper, to give a concise expression for various coefficients such as A_n, we have introduced the rule $0^0=1$.

The many-body Schrődinger equation for Ψ is

$$\begin{aligned} i\hbar\frac{\partial\Psi}{\partial t} &= \left[\sum_{i=1}^{N}\left(-\frac{\hbar^2}{2m}\nabla_i^2+V(\mathbf{r}_i)\right)\right. \\ &\quad \left.+g\sum_{i<j}^{N}\delta\left(\mathbf{r}_i-\mathbf{r}_j\right)-\mu\right]\Psi. \end{aligned} \tag{4}$$

Here μ is introduced due to the decay of η_1 and η_2. The introduction of μ is a direct generalization of the evolution equation for a single condensate with decay [18]. g represents the coupling due to interatomic collisions. To show clearly the role of complex potentials, we consider the situation of $g=0$. The role of interatomic collisions will be discussed at the end of this paper. Even the nonorthogonality between ϕ_1 and ϕ_2 is considered, it is not difficult to prove rigorously that the solution of Ψ is given by

$$\begin{aligned} i\hbar\frac{\partial\phi_1}{\partial t} &= \left(-\frac{\hbar^2}{2m}\nabla^2+V\right)\phi_1, \\ i\hbar\frac{\partial\phi_2}{\partial t} &= \left(-\frac{\hbar^2}{2m}\nabla^2+V\right)\phi_2, \\ i\hbar\frac{\partial A_n}{\partial t} &= -\mu A_n. \end{aligned} \tag{5}$$

We stress that the expression of Ψ given by Eq. (2) will not change in the whole dynamical process if ϕ_1 and ϕ_2 satisfy the above equation.

From Eq. (5), we have

$$i\hbar\frac{d\zeta}{dt}=\int\left(V-V^*\right)\phi_1\phi_2^*d\mathbf{r}. \tag{6}$$

As expected, $d\zeta/dt=0$ if V is a real potential. However, if V has an imaginary component, $d\zeta/dt$ may be nonzero. This means that with time evolution, ζ may be nonzero even for two initially orthogonal wave functions. This is the reason why we give a general consideration of ζ from the beginning. In this situation, the exact expression of the density expectation value is given by [20]:

$$n=a\,|\phi_1|^2+2b\times\mathrm{Re}\left[e^{i\varphi_c}\phi_1^*\phi_2\right]+c\,|\phi_2|^2, \tag{7}$$

where the coefficients are

$$\begin{aligned} a &= \sum_{i=0}^{\min(N_1-1,N_2)}\frac{A_n^2N_1!N_2!\,|\zeta(t)|^{2i}}{i!i!\,(N_1-i-1)!\,(N_2-i)!}, \\ b &= \sum_{i=0}^{\min(N_1-1,N_2-1)}\frac{A_n^2N_1!N_2!\,|\zeta(t)|^{2i+1}}{i!\,(i+1)!\,(N_1-i-1)!\,(N_2-i-1)!}, \\ c &= \sum_{i=0}^{\min(N_1,N_2-1)}\frac{A_n^2N_1!N_2!\,|\zeta(t)|^{2i}}{i!i!\,(N_1-i)!\,(N_2-i-1)!}. \end{aligned}$$

The relative phase φ_c is determined by $e^{i\varphi_c}=\zeta/|\zeta|$. The coefficient b shows directly the degree of quantum coherence between two initially independent condensates. The case $b/a<<1$ means a fragmented state. For b/a near 1, there is perfect quantum coherence between two initially independent condensates. One can prove that there is significant quantum coherence between two condensates for $N_1|\zeta|>1$ and $N_2|\zeta|>1$. In addition, b/a can be approximated well as 1 when $N_1|\zeta|>>1$ and

$N_2 |\zeta| >> 1$. A detailed analysis about the nonorthogonality on the quantum coherence can be found in Refs. [20].

Without the loss of generality, we consider the one-dimensional case and use the units where $\hbar = 2m = 1$. The initial wave functions of two condensates are assumed as $\phi_1(x, t=0) = \exp\left(-(x+s)^2/2\Delta^2\right)/\pi^{1/4}\Delta^{1/2}$ and $\phi_2(x, t=0) = \exp\left(-(x-s)^2/2\Delta^2\right)/\pi^{1/4}\Delta^{1/2}$. In our numerical calculations, $\Delta = 1$ and $s = 6$. This distance separation makes ζ between two initial condensates smaller than 10^{-15}. Based on Eq. (6), it is not difficult to prove that for uniform imaginary component in the complex potential, $\zeta(t)$ is always zero for two initially orthogonal wave functions. To make $\zeta(t)$ nonzero, nonuniform imaginary component is necessary. A natural choice is the following imaginary periodic potential [4, 5, 6, 7]

$$V = iV_0 \sin^2\left(\frac{2\pi x}{d}\right). \tag{8}$$

We calculate numerically the evolution of the system in this imaginary periodic potential. For the dynamical process of this initial condition, one can prove rigorously that the relative phase φ_c is always zero. In Fig. 2, we give $Log(|\zeta|)$ for different d and V_0 at $t = 5$. Because the loss of particles has been considered in the evolution of ϕ_1 and ϕ_2, the quantum coherence can be determined through the values of $N_1 |\zeta|$ and $N_2 |\zeta|$ with N_1 and N_2 being the initial particle numbers. Although $|\zeta| << 1$, the conditions $N_1 |\zeta| >> 1$ and $N_2 |\zeta| >> 1$ can be easily satisfied for a wide range of parameter d. Another unique behavior lies in that $|\zeta|$ changes significantly with different d. Experimentally, two equally polarized laser beams intersecting at an angle θ can be used to create the complex periodic potential with $d = \lambda/\sin(\theta/2)$ (λ is the laser wavelength). The inset in Fig. 2 gives $Log(|\zeta|)$ for $d = 5$ and different V_0. The exponential decay of η_1 with increasing V_0 is also shown in the inset.

For $\Delta = 1$, $s = 6$, $d = 5$ and $V_0 = 0.001$, we give in Fig. 3a the evolution of $\eta_1(t)$ and $|\zeta|$ based on the numerical calculations of Eq. (5). Because this value of V_0 is extremely small, the decay of η_1 is also very small. At $t = 5$, $\eta_1 = 0.995$ which agrees well with the approximate analytical expression $\eta_1 = e^{-V_0 t}$. This means that the loss of particles can be omitted, and thus the extra heating effect due to this imaginary periodic potential can be safely omitted in the whole dynamical process. As shown in Fig. 3a, we find that $|\zeta| << 1$. However, for this numerical result of $|\zeta|$, the conditions ($N_1 |\zeta| >> 1$ and $N_2 |\zeta| >> 1$) of ideal quantum coherence can be easily satisfied. In Fig. 3b, the evolution of b/a for $N_1 = N_2 = 10^5$ is shown. We see that for $t > 2.5$, there is very good quantum coherence between two initially independent condensates. This quantum coherence

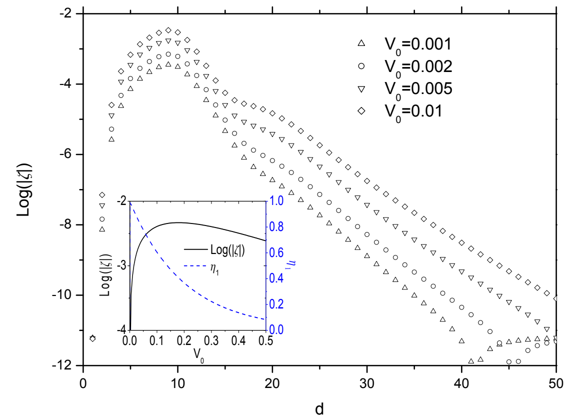

FIG. 2: Fig. a gives $Log(|\zeta|)$ for different d and V_0 at $t = 5$. The inset gives $Log(|\zeta|)$ and η_1 for $d = 5$ and different V_0. The units with $\hbar = 2m = 1$ are adopted.

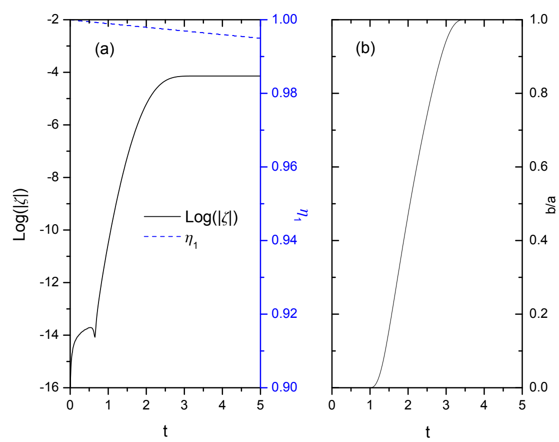

FIG. 3: Fig. a gives the time evolution of $Log(|\zeta|)$ and η_1. Fig. b gives the time evolution of b/a for initial particle numbers $N_1 = N_2 = 10^5$. It is shown that there is very good quantum coherence for $t > 2.5$, due to the presence of imaginary periodic potential. The units with $\hbar = 2m = 1$ are adopted.

originates from the nonzero $|\zeta|$ and exchange symmetry of identical bosons.

In Fig. 4a, we give further the evolution of $n(x,t)/(N_1 + N_2)$ for $N_1 = N_2 = 10^5$. Other parameters are the same as Fig. 3. Because of the quantum coherence between two condensates, obvious interference fringes are shown in the density expectation value, which is quite different from that of Fig. 4b with $V_0 = 0$. Because V_0 is much smaller than the kinetic energy $1/2\Delta^2$ of an atom, our numerical calculations show that the imaginary periodic potential will not play important role in the shape of $|\phi_1(x,t)|^2$ or $|\phi_2(x,t)|^2$. This is also the

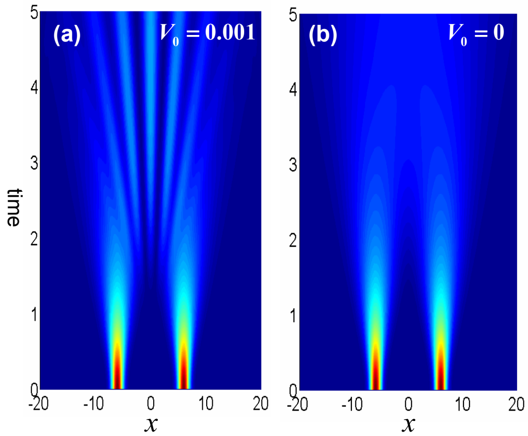

FIG. 4: Fig. a gives the evolution of $n(x,t)/(N_1+N_2)$ when the nonorthogonality due to imaginary periodic potential ($V_0 = 0.001$) and exchange symmetry of identical bosons are both considered. Fig. b gives the evolution of $n(x,t)/(N_1+N_2)$ without imaginary potential ($V_0 = 0$). The units with $\hbar = 2m = 1$ are adopted.

reason why $|\zeta| << 1$.

Without the imaginary periodic potential, one should note that after the overlapping between two initially independent condensates, there would be still interference fringes with completely random relative phase. This is due to the well-known measurement-induced interference mechanism [21]. In a single measurement, there will be clear interference fringes in the density distribution. By averaging the density distribution of a large number of measurements, however, there are no more interference fringes. This is significantly different from the situation that there is already quantum coherence between two condensates with time evolution, as shown in Fig. 4b. In this situation, we expect that there is no random shift in the interference fringes for different experiments with the same initial conditions. This provides a method to test the imaginary potential induced quantum coherence in a definite way.

In summary, the widely studied complex potentials for single-body problem are developed to many-body bosonic system. As a first step toward this new regime, we consider the role of complex potentials in the quantum coherence establishment process for two initially independent BECs. Besides the measure-induced interference mechanism [21] and interaction-induced quantum coherence [20, 22, 23, 24], a new coherence mechanism—imaginary potential induced quantum coherence is predicted theoretically. To experimentally test this imaginary potential induced quantum coherence, Feshbach resonance [25] can be used to rule out the interaction-induced quantum coherence by tuning $g = 0$. Even without complex potentials induced by the interaction between atoms and laser, there are various one-, two-, and three-body losses. It is possible that these losses can be described by a random imaginary potential. Our numerical calculations show that the inclusion of appropriate random imaginary potential can also induce the quantum coherence between initially independent BECs. This has potential application for the formation of a single condensate in the nonadiabatical evaporative cooling process, where initially a series of independent subcondensates will form. Considering the wide existence of random or regular complex potentials, the imaginary potential induced quantum coherence may have potential applications in diverse areas of physics where identical particle effect exists.

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